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AVERAGING VOLUME SIZE DETERMINATION OF ELECTROCONDUCTIVE POROSITY PROBES

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Abstract—Electroconductivity probes, used to measure local porosity in a packed bed or filter cake, have a localized averaging volume over which the measurement is made. The extent of this averaging volume is detected by the effect of the proximity of a multiphase boundary, such as the cake–slurry boundary in filtration, on a probe's measurement. A computer simulation and experimental verification show that the extent of the averaging volume is primarily dependent on the characteristic length between the electrodes. Probes can be designed by sizing the characteristic length to obtain averaging volumes which satisfy the REV criteria in volume-averaging theory. The extent of the averaging volume also determines how close a probe may be to a multiphase boundary without the boundary affecting the measurement.

Key Words: porosity, representative elementary volume, REV, porous media, conductivity, packed bed, filter cake

INTRODUCTION

The local averaging volume of a porosity probe is the spatial volume surrounding a porosity probe which is sensitive to changes in void fraction. This volume must be large enough to average the porosity measurement over all of the phases present, and yet it must be small enough that the measurement is local compared to the macro-scale dimensions of the multiphase system.

The size of the probe's averaging volume is needed to ensure that the averaged measurements satisfy the representative elementary volume (REV) criteria used in volume-averaging theory. It is also used to determine how close the probes may be to a multiphase boundary and still measure the local porosity with reasonable accuracy.

The purpose of this paper is to show that the averaging volume size can be determined for electrical conductivity probes which are used to measure porosity in flows through porous media. This averaging volume size is related to a characteristic length in two different probe geometries. By adjusting the characteristic length these porosity probes can be designed (sized) such that the averaging volume size is within the REV size criteria. Also, the characteristic length can be adjusted to allow measurements close to multiphase boundaries. Though only two probe geometries are considered, the method used here can be extended to other probe geometries and designs.

REV SIZE CRITERIA

The REV serves as a cornerstone in continuum modeling of transport phenomena in porous media (Bear & Bachmat 1984; Hassanizadeh & Gray 1979a). The volume-averaged approach to modeling porous media is well-known and need not be repeated here [see also, for example, Bachmat & Bear (1972), Carbonell & Whitaker (1984), Hassanizadeh & Gray (1979b, 1980) and Slattery (1972)].

The REV is defined such that no matter where the REV is located in a two-phase porous media system it contains a statistically meaningful distribution of both phases. Also, the REV is defined such that it is within a size range in which the averaged properties are independent of the REV size. Outside of this size range the averaged properties may be dependent on the averaging volume size and geometry.

Celmiņš (1988) shows that to satisfy the REV criteria for particle–gas flow in a tube the averaging volume must be less than the size of the flow structure being investigated and must be large enough to average over at least 60–150 particles. For a spherical averaging volume the minimum size corresponds to at least 4–5 mean distances between particle centers.

Bachmat & Bear (1987) show that for a cubical REV the minimum size is given by

$$l_{\text{min}} = 20C_{\Delta}\Delta, \qquad [1]$$

where l_{min} is the length of one side of the minimum size cube and $C_{\Delta}\Delta$ is the length of a cubic subdomain. The length of the cubic subdomain can be roughly interpreted to be the mean distance between particle centers. They also determine the maximum REV size to be given by

$$l_{\text{max}} = \frac{2\epsilon|_{x_0}}{|\text{grad}\,\epsilon|_{x_0}}\delta, \qquad [2]$$

where l_{max} is the length of one side of the maximum size cubical REV; ϵ is the porosity, δ is an arbitrarily selected small number in the range 0–1 which represents an acceptable average relative error introduced by replacing the porosity, ϵ, with $\epsilon|_{x_0}$; and $|_{x_0}$ indicates that the quantity is evaluated with the averaging volume centered at position x_0.

Equation [2] shows that the maximum size of the REV for porosity measurement is inversely proportional to the gradient of the porosity. This means that smaller averaging volumes are required when the porosity is non-uniform. It is possible in some systems with large porosity gradients for l_{min} to be larger than l_{max}, in which case the REV does not exist.

The above criteria for the maximum and minimum size limitations on the REV are determined for spherical and cubical averaging volumes. The averaging volumes of instrument probes in general may be of other geometric shapes. As long as the probe averaging volume is well within the criteria for the spherical and cubical geometries then it is reasonable to assume that the probe averaging volume is within the REV size range.

BOUNDARY PROXIMITY EFFECTS

In cake filtration the boundary between the cake and the approaching slurry is characterized by a discontinuity in the solid phase concentration. When an electrical conductivity probe, used to measure the porosity within the cake, is positioned near the cake–slurry boundary part of the electrical current can follow a path of lesser resistance through the slurry phase, as shown in figure 1. This results in an inaccurate measurement of the local porosity. Such concerns have been expressed by Shirato & Aragaki (1972), Shirato *et al.* (1971) and Wakeman (1981), though no previous attempts have been made to quantify the effect.

Clearly, the problem of the cake-slurry boundary affecting the porosity measurements is related to the size of the probe's averaging volume. When the cake–slurry boundary is within the probe's averaging volume then the boundary causes the measurement to be in error. If the probe's averaging volume size is known then measurements too close to the boundary can be avoided.

Electrical conductivity measurements have been used in other processes, such as fluidized beds (Begovich & Watson 1978; Turner 1976). In such processes similar boundary proximity effects can occur at a bed inlet and outlet.

Other processes also use electrical conductivity measurements, such as slurry flows through tubes (Merilo *et al.* 1977; Nasr-El-Din *et al.* 1987). In these processes the solids concentration is continuous and the boundary effects described above do not apply. However, Nasr-El-Din *et al.* (1987) show that the proximity of the tube wall to the radical location of the porosity probe can affect the probe measurement, which indicates that the probe's averaging volume may fill the entire cross-sectional area of the tube. They are still able to obtain local measurements in the radial direction because the probes are most sensitive to the slurry closet to the probe and they are able to take the radial position dependence into account in the probe calibration.

DETERMINATION OF THE PROBE AVERAGING VOLUME SIZE

Two probe designs are considered here in which the electrodes are flush-mounted in a filter assembly wall. The boundary proximity effect is used to determine the size of the averaging volume

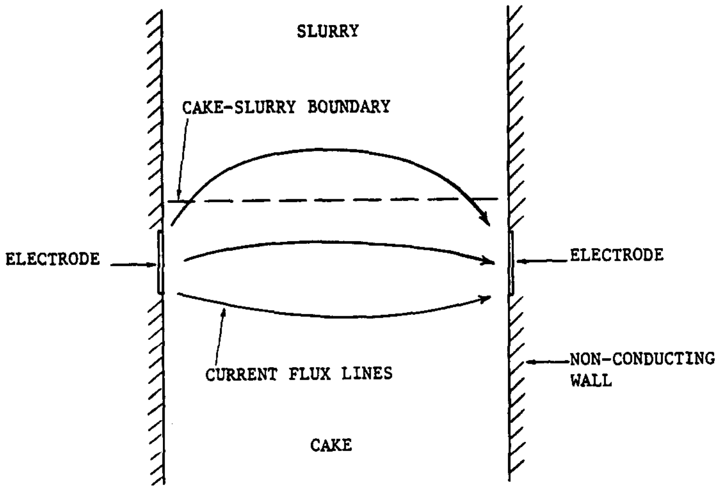

Figure 1. A porosity probe (electrode pair) positioned near a cake–slurry boundary. The electric current flux lines bend towards the slurry with its lower electrical resistance, which results in an inaccurate measurement.

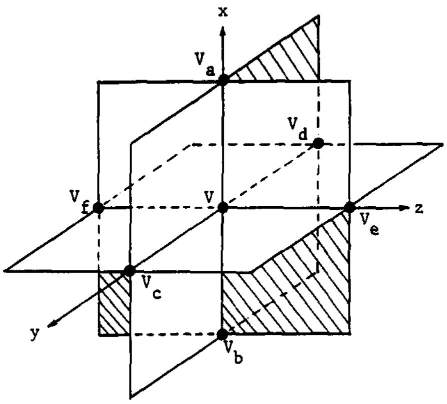

Figure 2. Three-dimensional rectangular grid for the computer simulation. Shown here are the potentials at the grid points surrounding an arbitrary grid point which has a potential of V.

in the direction normal to a multiphase boundary such as the cake–slurry boundary in filtration. The method can easily be extended to the other dimensional directions of the averaging volume which are not considered here.

Only flush-mounted probes with two electrodes are considered here because they do not intrude or obstruct the flow, or in this case, the formation of a filter cake. Nasr-El-Din *et al.* (1987) point out that such designs have the disadvantage that the distribution of porosity over the cross section cannot be measured. In filtration the porosity varies primarily in the direction of the flow and not over the cross-sectional area, hence these probes are adequate for that purpose.

Nasr-El-Din *et al.* (1987) also point out that probe designs with two electrodes, which serve the dual purpose of generating the electric potential field and sensing the changes in conductivity, are affected by the fluid flow rate. In filtration the flow rates are slow enough that this is not a significant factor.

Computer simulation

Empirical determination of the axial dimensions of the probe averaging volume size as a function of probe geometry and material properties would require numerous experiments. To reduce the experimental effort the system is numerically modeled and the results are verified with one experiment on each probe design.

In this simulation the cake and the slurry systems are modeled as separate systems that interact across the cake–slurry boundary. The porosity of each system is assumed to be uniform, which makes the conductivity of each system uniform. A discontinuity in the conductivity occurs at the cake–slurry boundary due to the porosity discontinuity between the systems. The conductivity discontinuity affects the probe measurement when the boundary is within the probe's averaging volume.

Treating the multiphase mixture as a pseudo-single phase and assuming alternating current and low iron concentration, the ion species balance in a system of uniform conductivity reduces to Laplace's equation in the voltage potential, V:

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0 \qquad [3]$$

(Kroyta *et al.* 1970).

The multiphase mixture can be treated as a pseudo-single phase relative to the voltage potential field when the porosity probes satisfy the REV criteria. The REV criteria assures that the particles are sufficiently smaller than the dimensions of the probe and that a sufficient number of particles are present such that at the volume-averaged scale the effect of the particles on the potential field is smoothed out and the presence of the particles only affects the measured conductivity.

In finite-difference form, Laplace's equation gives a second-order approximation for the potential at a given point in space as

$$V = \tfrac{1}{6}(V_a + V_b + V_c + V_d + V_e + V_f) \qquad [4]$$

(Carnahan *et al.* 1969), where the potentials on the r.h.s. of the equation are the potentials at the surrounding grid points, as shown in figure 2.

The potentials of the electrode surfaces are assumed to be constant values, while all of the other surfaces in the simulation volume are assumed to be non-conducting. The potential gradient across a non-conducting surface is zero and this is modeled by the mirror-image method. The mirror-image method, for example, sets the potential V_c equal to V_d at a non-conducting surface normal to the y-axis and the potential at the surface is calculated using [4].

At a cake–slurry boundary, where the conductivity is discontinuous, a jump balance sets the current flux into the boundary from the cake system equal to the flux entering the slurry system. For such a boundary normal to the x-axis this is mathematically stated as

$$\sigma_C \left.\frac{\partial V}{\partial x}\right|_C = \sigma_S \left.\frac{\partial V}{\partial x}\right|_S, \qquad [5]$$

where the subscripts C and S correspond to the cake and slurry systems, respectively.

In finite-difference form [5] models the potential at the cake–slurry boundary by

$$V = \frac{\sigma_C V_C + \sigma_S V_S}{\sigma_C + \sigma_S}. \qquad [6]$$

The procedure for modeling a particular probe and system geometry is to define a three-dimensional grid with equal distances between the grid points in each direction, such that the boundaries of the cake and slurry systems correspond with a planar surface in the rectangular grid. The electrode potentials are assigned constant values (1 and − 1 V) and all the remaining grid points are assigned zero potential values. Using a Gauss–Seidel method (Carnahan *et al.* 1969), the potentials at each point are iteratively calculated using [4] and [6] until the changes in the calculated values become insignificantly small.

Probe designs

Three-dimensional and side views of the simulation volumes for opposing and planar probe designs are shown in figures 3 and 4. These figures indicate the characteristic dimension, Y_c, which is the distance between the electrodes. Figure 3 shows the height and width dimensions, h and w, of the electrodes and figure 4 shows the distance between the edge of the electrodes and the cake–slurry boundary, X. In the simulations X is varied to determine the proximity effect on the measurements.

Mathematically, the simulation volumes are modeled by a three-dimensional rectangular grid over which the potential at each grid point is determined using the Gauss–Seidel method. The simulation volumes are made large enough that the boundaries of the simulation volume do not affect the measured results.

A typical conductivity meter determines the measured conductivity from the expression

$$\sigma_{\text{meas}} = \mathrm{K}\frac{I_{\text{meas}}}{\Delta V}, \qquad [7]$$

where ΔV is the constant voltage potential difference between the electrodes, K is a constant geometric factor for the particular probe design and I_{meas} is the measured total current through the electrodes.

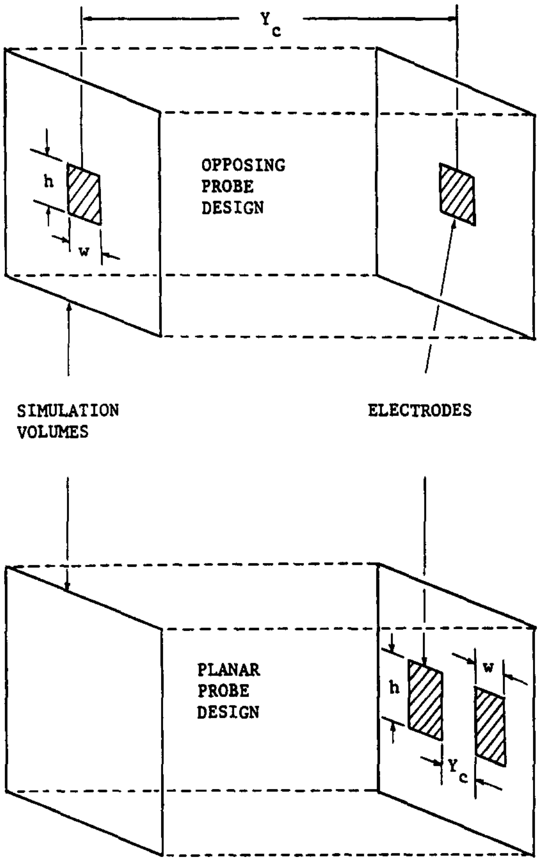

Figure 3. Comparison of the opposing and planar conductivity probe designs. The characteristic distance, Y_c, is the distance between the electrodes.

In the simulation the total current, I_{meas}, is calculated from

$$I_{\text{meas}} = \int_A \sigma \frac{\partial V}{\partial y} \, \mathrm{d}a \qquad [8]$$

by using the trapezoidal rule to numerically evaluate the integral over a convenient planar surface separating the two electrodes.

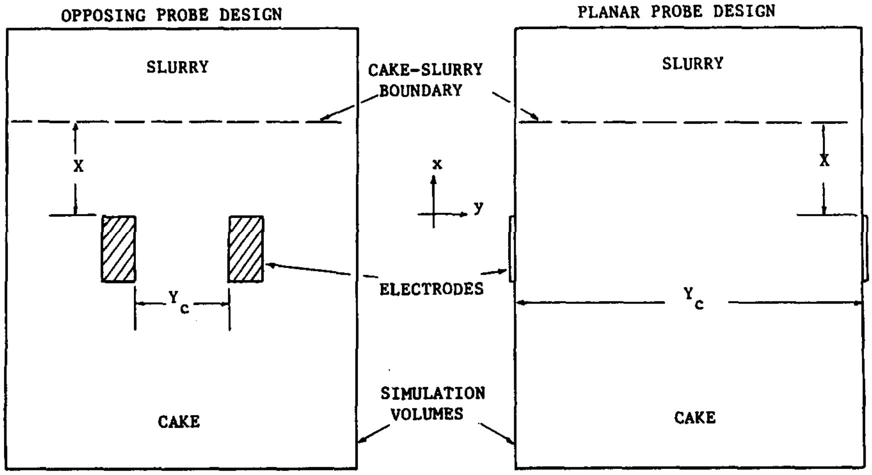

Figure 4. Side views of the simulation volumes for the opposing and planar probe designs. Across the cake–slurry boundary is a jump in electroconductivity. X is the distance between the near edge of the electrode pair and the boundary.

To compare the results between the different simulations a dimensionless conductivity is defined as

$$\sigma' \equiv \frac{\sigma_{\text{meas}} - \sigma_C}{\sigma_S - \sigma_C}, \qquad [9]$$

where σ_C and σ_S are the measured conductivities when only the cake is present or only the slurry is present in the simulation volume. With this definition, the dimensionless conductivity is zero when the cake–slurry boundary does not affect the measurement.

Combining [7] and [9] gives

$$\sigma' = \frac{I_{\text{meas}} - I_C}{I_S - I_C}, \qquad [10]$$

which relates the dimensionless conductivity to the total currents determined in the simulation.

Simulation results

The results of the computer simulation are summarized in figure 5. In this figure data points are plotted which relate the dimensionless conductivity, σ', to the dimensionless distance between the electrodes and the cake–slurry boundary, X/Y_c. The results show that the dominant effect is the probe geometry and the characteristic length, Y_c. The ratio of the conductivities, σ_S/σ_C, and the dimensionless height and width, h/Y_c and w/Y_c, affect the results to a lesser extent.

The plots in figure 5 show that a dimensionless conductivity approaches zero for the opposing probes at a dimensionless position of $X/Y_c \cong 0.25$ and for the planar probes at a dimensionless position of $X/Y_c \cong 1.0$. Based on this formation a critical distance, X_c, is defined for the two probe designs as

$$X_c \equiv \begin{cases} 0.25Y_c & \text{opposing probe} \\ Y_c & \text{planar probe,} \end{cases} \qquad [11]$$

which is the distance from the edge of the electrode pair to the edge of the probe's averaging volume. Due to symmetry, the averaging volume extends the same distance above and below the electrodes. The length of the averaging volume in the x-direction is

$$l_x = h + 2X_c. \qquad [12]$$

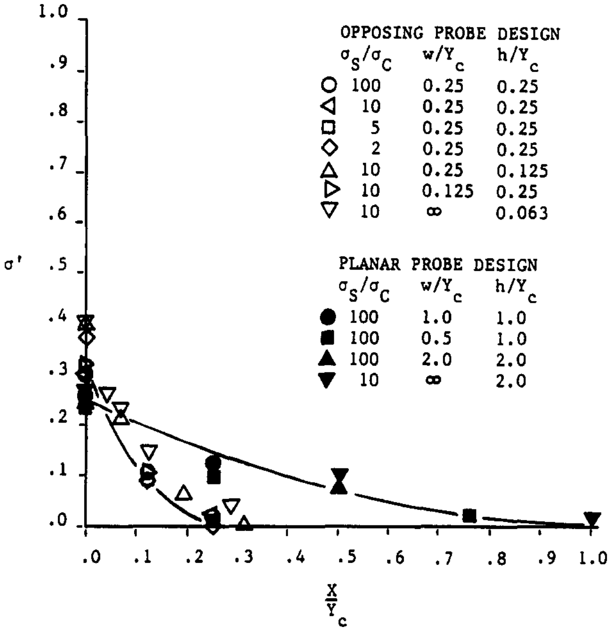

Figure 5. Proximity effect of the cake–slurry boundary on the dimensionless conductivity as determined in the computer simulation for the opposing and planar probe designs. The values in the symbol key are the conductivity ratio, dimensionless width and dimensionless height conditions for which the data is calculated.

In the design of porosity probes for a particular purpose, the averaging volume length, l_x, must satisfy the REV criteria, such as that given by [1] and [2]. In sizing the electrodes to make measurements close to a multiphase boundary, such as the cake–slurry boundary in filtration, the closest the probes can be to the boundary is given by X_c. The results of the simulation above show that the lengths, l_x and X_c, can be designed into the probe by sizing the characteristic length, Y_c.

EXPERIMENTAL VERIFICATION

The simulation results provide a basis for designing porosity probes for making local measurements in a particular system. After fabrication, such probes should be tested and the averaging volume size verified. This is done here with a planar and an opposing probe design.

Probe designs

The system tested here is a packed bed of glass beads having mean particle dia 0.2 mm. From [1], the minimum averaging volume cubical length that fits the REV criteria is on the order of 4 mm. The packed bed is expected to have a uniform porosity for which [2] indicates that the REV criteria has no maximum size. Such a packed bed as this allows the averaging volume size to be experimentally measured by the same technique as in the simulation by varying the probe's proximity to the boundary of the top of the packed bed.

The opposing probe is constructed of silver-plated copper disk electrodes with 6.35 mm dia. These disks are mounted flush and diametrically opposed in a 102 mm dia. Plexiglas cylinder. From [11] the critical distance for this design, X_c, is about 25 mm and from [12] the averaging volume length, l_x, in the x-direction is 56 mm.

The planar probe is constructed of silver wire electrodes 3 mm high, 1 mm wide and with a characteristic length of 3 mm. These electrodes are mounted flush in the same Plexiglas cylinder as is used with the opposing probes. From [11], the critical distance for this probe is 3 mm and from [12] the length of the averaging volume in the x-direction is 9 mm. The lengths of the averaging volumes in the x-direction for both probes are sufficiently larger than the REV criteria minimum cubical length that, even though the averaging volumes are not cubical, they are within the REV size range.

Experimental method and results

In the experiment the slurry is replaced with a 0.0025 M solution of potassium chloride in deionized water. The cake is replaced with a packed bed of the glass beads in which the void space is filled with the same water solution. The boundary between the packed bed and the water solution above the bed is moved relative to the porosity probe positions by adding incremental layers of glass beads to the top of the cake.

The data points from the experiment are plotted in figure 6 along with the curves from the simulation shown in figure 5. The dimensionless conductivity approaches zero at a dimensionless position of about 0.2 for the opposing probe and about 1.0 for the planar probe, which agree well with the simulation. The discrepancy between the simulation curve and the experimental data for the opposing probe design is probably due to the difference between the rectangular geometry used in the simulation and the cylindrical geometry used in the experiment. With the planar probe the change in geometry is insignificant.

The critical distance, X_c, and the length of the averaging volume in the x-direction for these probes satisfy the REV criteria for the glass beads. The planar probes are not affected by the cake–slurry boundary when the boundary is at least 3 mm from the electrodes and the opposing probes are not affected when the boundary is at least 20 mm away from the electrodes.

Probe calibration

Meredith & Tobias (1962) and Nasr-El-Din *et al.* (1987) review a number of correlations between porosity and electrical conductivity. The intent here is not to develop a new correlation but to

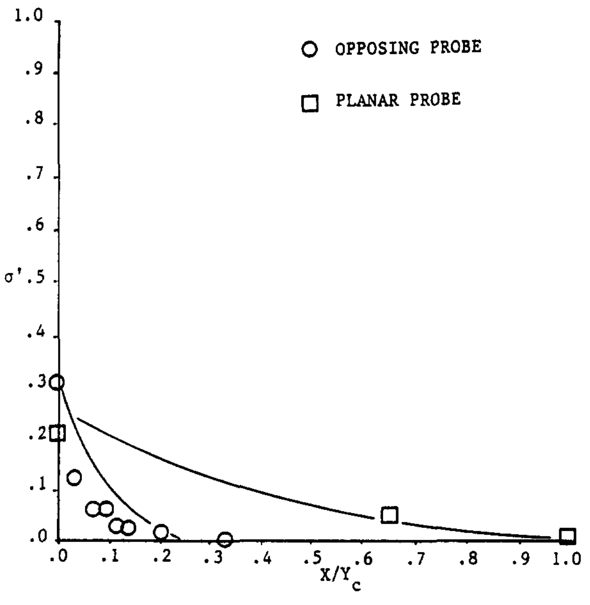

Figure 6. Proximity effect of the boundary on the dimensionless conductivity as measured in the glass bead packed bed experiment for the opposing and planar probe designs. The solid curves are from the computer simulation results in figure 5.

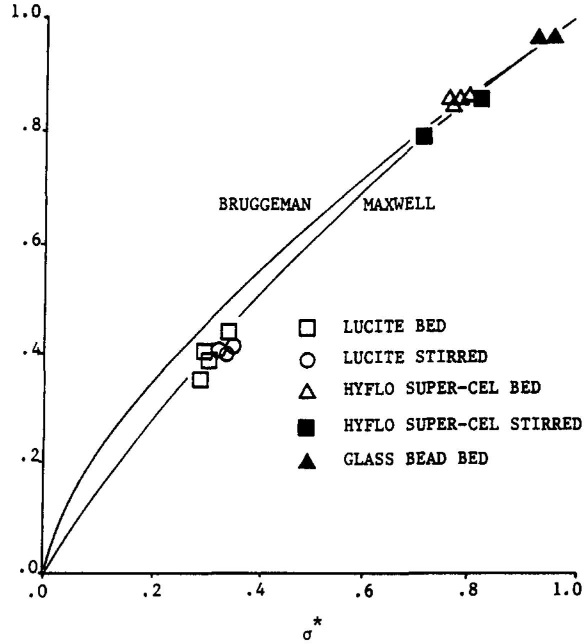

Figure 7. Plot of porosity vs dimensionless conductivity data measured with the planar electrode and predicted by the Maxwell and Bruggeman correlation curves. The particles are suspended by stirring or are gravity settled into packed beds, as indicated in the symbol key.

compare data from the planar probe design with the existing Maxwell and Bruggeman correlations, given by

Maxwell,
$$\sigma^* = \frac{2\epsilon}{(3-\epsilon)} \qquad [13]$$

and

Bruggeman,
$$\sigma^* = (\epsilon)^{1.5}, \qquad [14]$$

for a non-conducting solid particle phase. The dimensionless conductivity in these correlations is defined as

$$\sigma^* \equiv \frac{\sigma_{\mathrm{meas}}}{\sigma_{\mathrm{fluid}}}, \qquad [15]$$

where σ_{meas} is the measured conductivity of the multiphase mixture and σ_{fluid} is the measured conductivity of the particle free fluid.

The experimental data is plotted in figure 7, which shows that for the planar probe design the Maxwell correlation fits the experimental data more closely in the lower porosity range than the Bruggeman correlation. In the high porosity range both correlations fit the data equally well.

The experimental data in figure 7 is obtained from packed beds of known total bed volumes and known solid phase particle volumes. The porosity probes are positioned in the bed such that the bed boundaries are outside the range of the probe's averaging volume. Also, as indicated in figure 7, experimental data are plotted for measurements in high porosity systems in which the solid particles are well-dispersed by stirring.

The porosity plotted in figure 7 is calculated as the void fraction, the volume of the fluid phase in the packed bed or stirred system divided by the total volume of the packed bed or stirred system. The dimensionless conductivity is determined from [15], with separate measurements of the conductivity of the fluid phase and the conductivity of the packed bed or stirred vessel at the same temperature and same fluid phase ionic concentration.

CONCLUSIONS

The results of this work show that the length of the averaging volume of an electrical conductivity probe in the x-direction normal to a cake–slurry boundary can be measured by the effect of the boundary on the measurements. The computer simulation and experimental verification show that the opposing and planar probes can be designed by selection of the characteristic length, Y_c, to fit the REV criteria and distance to a cake–slurry boundary for a specific process.

The method of detecting the effect of a cake–slurry boundary on the probe measurements can be extended to determine the extent of the probe's averaging volume in the y- and z-directions which are not considered in this work. Also, this work can be extended to other probe designs such as those which may be used to measure the radial dependence of the local porosity within a packed bed or filter cake instead of only near the walls.

The computer simulation here only considers systems of uniform porosity. When the porosity has a gradient, then the REV criteria in [2] limits the maximum size of the allowable averaging volume size. The averaging volume size may change in a system with a porosity gradient but the change is probably not extreme. Additional work is needed in this area to determine how much the averaging volume size is affected by a porosity gradient.

In processes in which the REV criteria cannot be satisfied, such as when the gradient in the porosity is so large that $l_{min} > l_{max}$, or when the porosity is so high that l_{min} exceeds the macro-scale dimensions of the system, then strong perturbations can occur in the measurements. In such a system the results of this work are not applicable.

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